Exercise 1: Pressure Sensor

This problem set is intended to be a walk through the design and operation of a silicon pressure sensor. The pressure sensor is made of highly boron doped resistors standing on a silicon oxide film to eliminate the p-n junctions and therefore increase the operating temperature range of silicon based pressure sensors as shown in Figure 1.

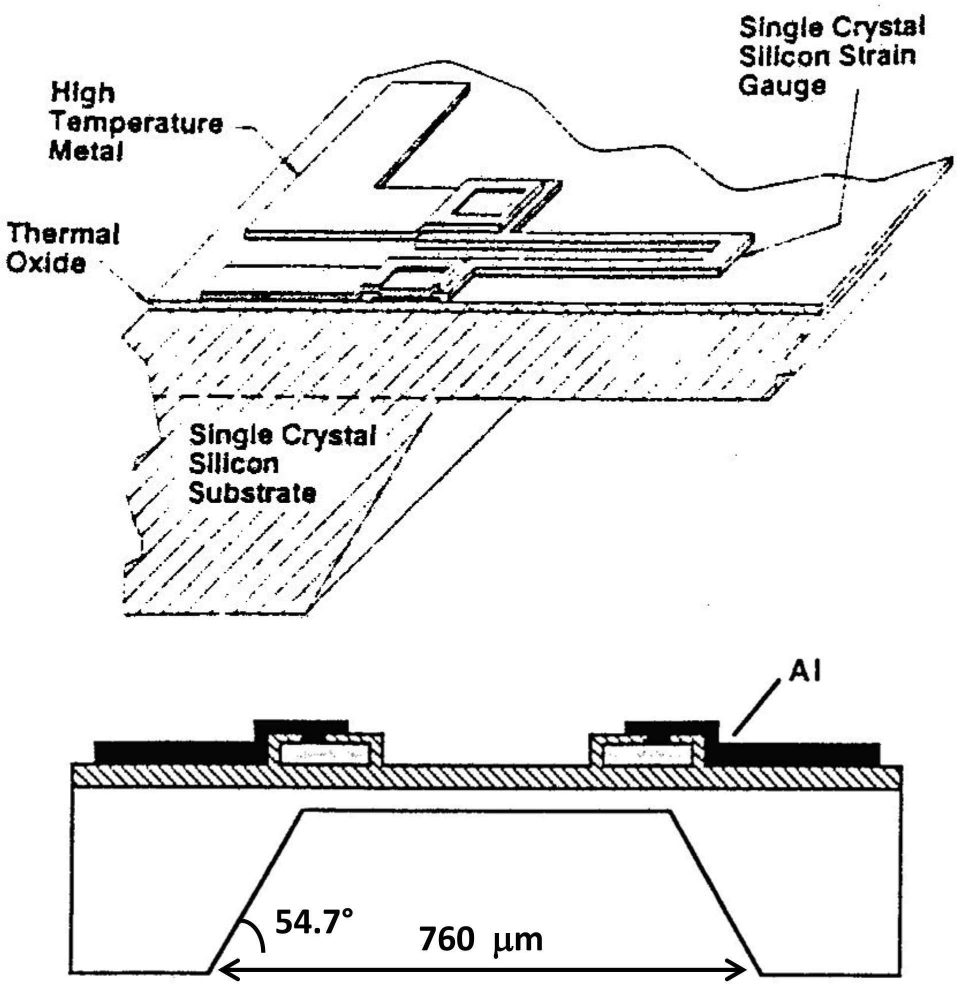


Figure 1: Cutaway view and cross-section of single-crystal silicon resistors standing on a silicon membrane passivated with a silicon oxide film.

Some characteristics:

- Wafer thickness of 275 μm.
- The silicon membrane has a thickness of 20 μm.
- The piezoresistors thickness is 1.0 μm.

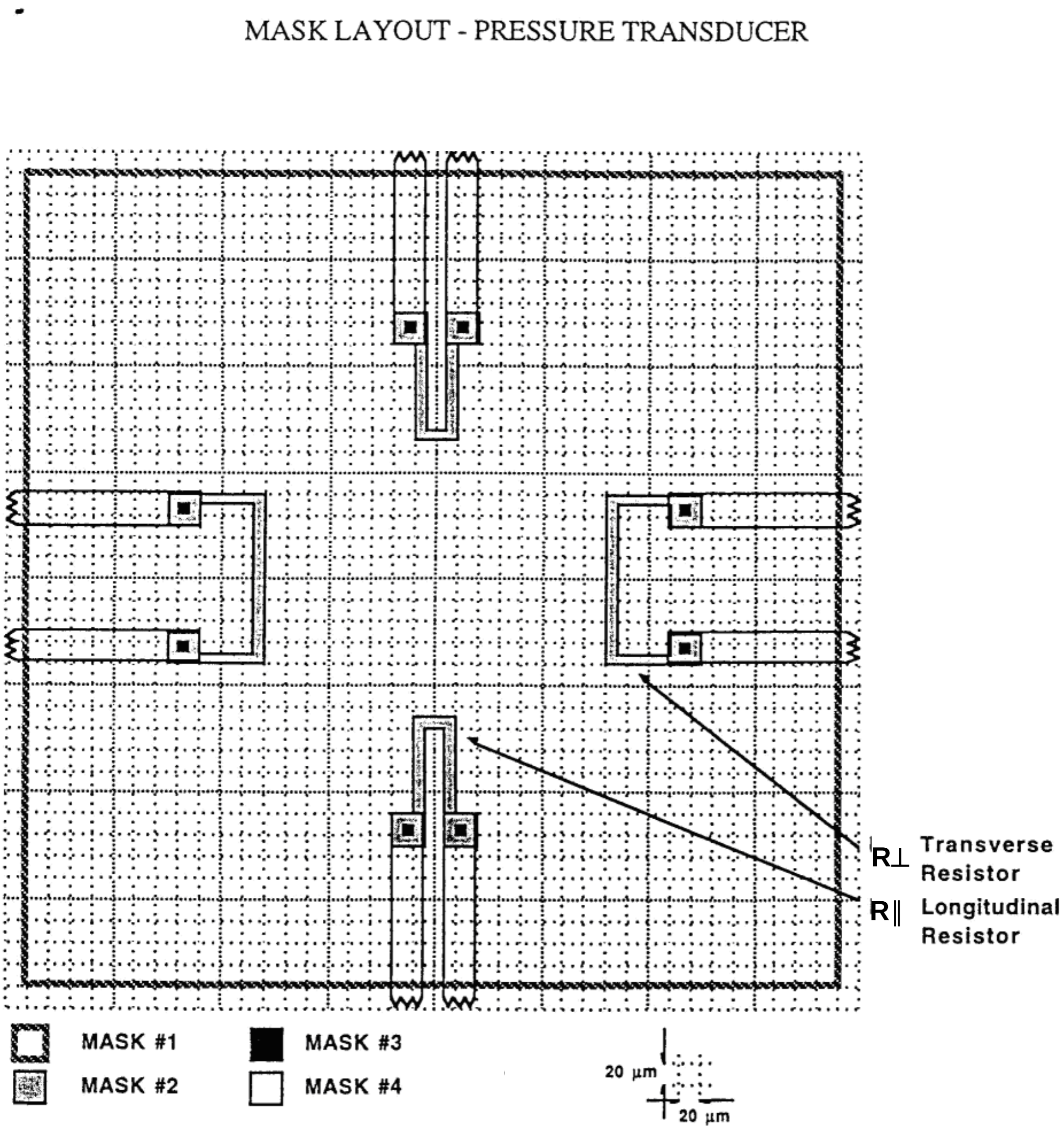


Figure 2: Layout of the pressure sensor using a 4 masks fabrication process.

a) What is the criteria to define the location of the piezoresistors on the silicon membrane ?

Answer: Region with the higher stress. For a clamped square membrane, this region is near the edges of the membrane.

b) Why are the piezoresistors positioned in two different orientations, $R_{\perp}$ and $R_{\parallel}$.

Answer: To have opposite variations of their resistance value for implementation in a Wheatstone bridge for the read-out circuit.

c) On Figure 2 draw the edges of the silicon membrane.

Answer: Membrane is a 400 x 400 um^2 square with the piezoresistors on the inner side of the membrane and their contacts on the outside.

d) Complete the section below on the characteristics of the pressure sensor fabricated using the mask layout presented in Figure 2. (thickness of piezoresistors: 1 μm, ρ Si = 0.001 Ω-cm).

Membrane thickness: 20 μm Membrane edge length: 400 μm

Sheet resistance of the silicon used for the Si piezoresistors: 10 Ω/□
Sheet resistance = 0.001 Ω*cm / 10^{-4} cm

Values of the two transversal and longitudinal piezoresistors:

$R_\perp$ = 190 Ω (18 squares + 4 × ¼ square for the corners)
$R_\parallel$ = 250 Ω (24 squares + 4 × ¼ square for the corners)

e) Using this pressure transducer, we will determine the expected resistor change due to an applied pressure. Some expressions are given to you below to solve the following questions.

For silicon: $\frac{E}{1-\nu^2} = 200$ GPa,

For a square diaphragm, the deformation at the centre is:

$$w = 1.638 \times 10^{-3} \frac{12(1-\nu^2)}{E} \bullet \frac{l^4}{h^3} P$$

$$\sigma_\perp = \frac{0.294 \times l^2 \times P}{h^2} \qquad \sigma_\parallel = \frac{0.115 \times l^2 \times P}{h^2}$$

$$\frac{\Delta R_\parallel}{R_\parallel} = \pi_\parallel \sigma_\parallel + \pi_\perp \sigma_\perp \approx {}^1/_2\, \pi_{44} (\sigma_\parallel - \sigma_\perp)$$

$$\frac{\Delta R_\perp}{R_\perp} = \pi_\parallel \sigma_\perp + \pi_\perp \sigma_\parallel \approx {}^1/_2\, \pi_{44} (\sigma_\perp - \sigma_\parallel)$$

Assuming an applied pressure of 2.5 MPa and ½(π_{44}) = 36x10^{-11} Pa^{-1}:

i) What is the deflection in the middle of the diaphragm?

Answer: w = 0.786 um

ii) Determine the longitudinal and the transverse stress at the centre of the diaphragm edge where the piezoresistors are located.

Answer: $\sigma_{\parallel}$ = 115 MPa $\sigma_{\perp}$ = 294 MPa

iii) What is the fractional change in resistance for a 2.5 MPa load for resistors placed in parallel and perpendicular to the diaphragm edge?

$\Delta R_{\parallel} / R_{\parallel} = -0.064 = \alpha$ $\Delta R_{\perp} / R_{\perp} = 0.064 = \beta$

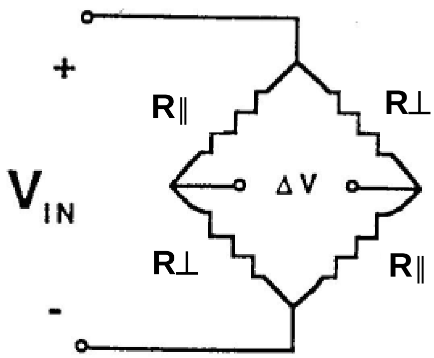
f) Assume that the resistors are configured as a full Wheatstone bridge (see drawing below).

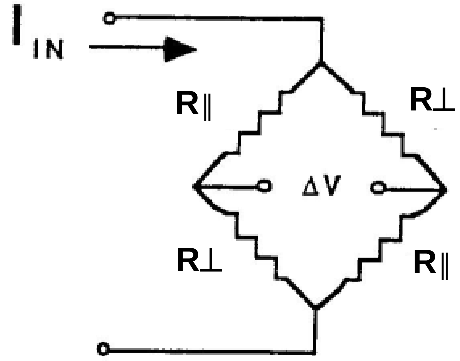
i) Determine an expression for the differential output voltage from this circuit for a change in pressure, **P**, as a function of RII and R⊥. Derive the expression for both constant voltage drive and constant current drive, as shown below.

$$Z = (R_{\perp} - R_{\parallel})/2$$

Answer: Z: Bridge's input impedance

$$\Delta V = \left(\frac{R_{\perp} - R_{\parallel}}{R_{\perp} + R_{\parallel}}\right) V_{IN} \qquad \Delta V = (R_{\perp} - R_{\parallel}) I_{IN}/2$$





ii) Is either mode, constant current / constant voltage, preferred?

Answer: Contact voltage non-linear relationship, constant current has the advantage to result in a linear relationship.

iii) Using the resistance values from the previous questions, what is the output voltage for a 2.5 MPa load? Assume that $\mathbf{V_{IN}}$ and $\mathbf{I_{IN}}$ are chosen such that the static power dissipation is less than 10 mW.

Answer: $10\ mW = Z \times I_{IN}^2$ $I_{IN} = 6.75$ mA and $V_{IN} = 1.5$ V

$R_{||} = R_{||,0}(1+\alpha)$ and $R_{\perp} = R_{\perp,0}(1+\beta)$

$\Delta V_{out} = \Delta V_{(2.5\ MPa)} - \Delta V_{(0\ MPa)}$

$\Delta V_{out} = [R_{\perp,0}(1+\beta) - R_{||,0}(1+\alpha)]\times I_{IN}/2 - [R_{\perp,0} - R_{||,0}]\times I_{IN}/2$

$\Delta V_{out} = [R_{\perp,0}\times\beta - R_{||,0}\times\alpha]\times I_{IN}/2 = 95$ mV for 2.5 MPa = 0.038 μV / Pa

iv) Estimate the minimum detectable pressure if the dominant noise source is thermal noise in the resistors. Assume a bandwidth of 10 kHz. $\nu = \sqrt{4kTR\Delta f}$

Answer: $R = 250\ \Omega$ $\Delta f = 10$ kHz $T = 23°C \Rightarrow \nu = 0.2\ \mu V$

Minimum detectable voltage is defined as twice the noise: 0.4 μV which corresponds to 10 Pa (using the answer in question f)iii above.

g) Estimate the effect of the following variables on the sensitivity of the sensor.

i) $\pm$ 5 μm wafer thickness variation.

Answer: In equation of σ, variation of thickness of 5 μm on a total membrane thickness of 20 μm represents a change of about a factor 2 for σ. This results in a variation of $\pm\sim$ 50% of change in sensitivity.

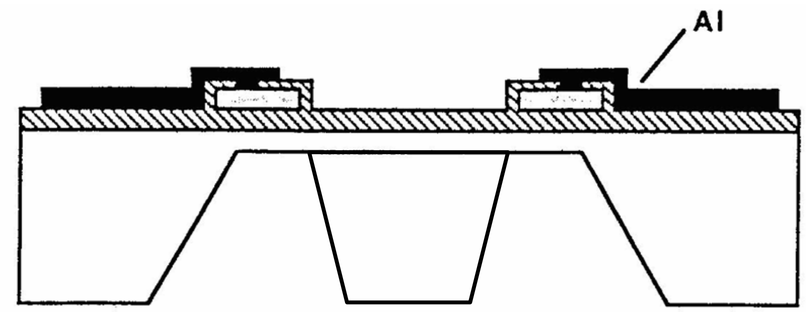
ii) $\pm$ 1 μm line width variation.

Answer: This will cause a $\pm$ 10% variation of the piezoresistors values resulting in a $\pm$ 10% variation in sensitivity.

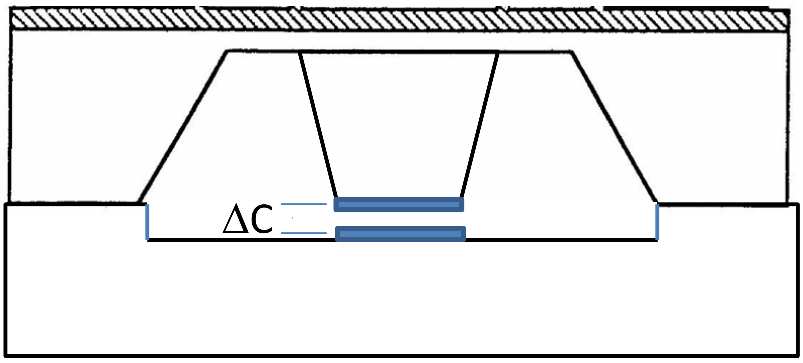
Exercise 2: Accelerometer

a) Using the piezoresistive principle, what would you change to the configuration of the pressure sensor in exercise one to make a piezoresistive accelerometer?

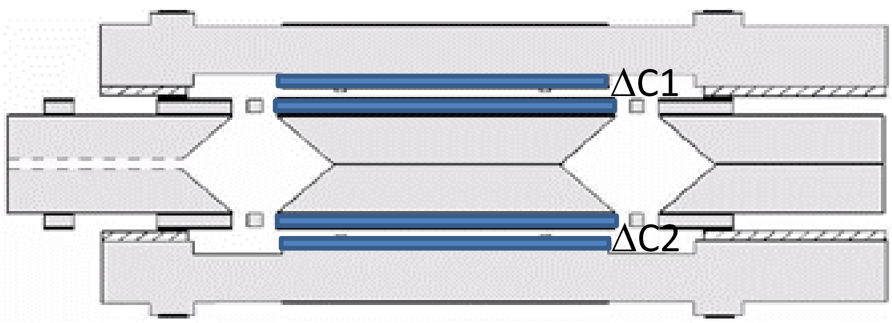
Answer: Add a proof mass for higher sensitivity (higher force and higher stress on the piezoresistors area).



b) And using the same configuration as in a) how would you implement capacitive transduction? And if you would like to perform a differential capacitive measurement?

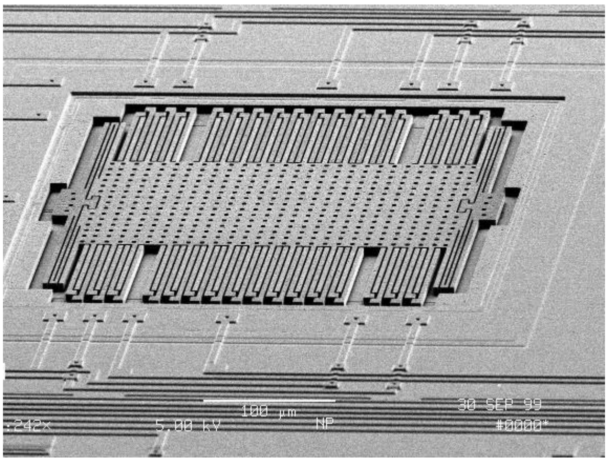


Differential configuration: Capacitive accelerometer from Colibrys



c) What is currently the mainly used capacitive transducer configuration in MEMS accelerometers?

Answer: Comb electrodes for capacitive differential sensing. There is a set of stationary fingers and a set of fingers that move under acceleration. See slides 25 to 32 from Lecture 2 on MEMS sensors part1.



Exercise 3: Gyroscope

a) Describe the implementation of a MEMS gyroscope using a schematic drawing.

Answer: See slides 73 to 79 from Lecture 2 on MEMS Sensors part1.

b) Give the name and the expression of the force that will result in a displacement of the structure due to an angular rotation.

Answer: Coriolis force with m: mass, v: speed of the mass, Ω: angular rate of rotation

$$\vec{F}_C = 2\,m\,(\vec{v} \times \vec{\Omega}) = -2\,m\,(\vec{\Omega} \times \vec{v})$$

c) Which parameters can you play with to increase the sensitivity of the gyroscope response? And how these should be optimised for higher sensitivity.

Answer: Weight of the masses and their velocity. Increase of mass and increase of velocity (increase of the driving frequency of the comb drive actuation of the masses)